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TIME DEPENDENCE OF THE HEAT-TRANSFER COEFFICIENT BETWEEN COMPONENTS OF A COMPOSITE DURING HEAT TRANSFER

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The effect of the thermophysical and geometrical characteristics of the components of a composite on the dynamic behavior and asymptotic value of the coefficient of heat transfer between the layers is studied.

A multitemperature approach [1-3] based on averaging of the temperature fields of each component within an elementary microvolume is being employed increasingly in the calculation of the thermal state of heterogeneous media. In the case of layered and reinforced media this makes it possible to reduce the dimension of the initial heat equations, thus greatly facilitating the solution of the problem. The resulting system of differential equations (the order of the system is equal to the number of components) is closed by introducing a relation between the density of the thermal flux between the components and their average temperatures. In [1] such a relation was obtained from phenomenological linear relations between the thermodynamic forces and fluxes:

$$q_{ij}=\alpha(\hat{T}_i-\hat{T}_j). \tag{1}$$

It is understood that α is an effective characteristic of the thermophysical and geometric parameters of the structure of the composite. The explicit form for α for a layered composite was obtained in [2] and [3], respectively, as

$$\alpha_2=2\sqrt{3}\frac{l_1l_2\lambda_1\lambda_2}{l_k^2(l_1\lambda_1+l_2\lambda_2)},\quad \alpha_3=\frac{3\lambda_1\lambda_2}{l_1\lambda_2+l_2\lambda_1}. \tag{2}$$

The heat-transfer coefficient α is an integrated characteristic of the rate of heat transfer between the components. The integrated heat-transfer characteristics are generally not constants. It is known [4], e.g., that the effective thermal-conductivity coefficient, which is also an integrated characteristic, depends on time. By analogy we can assume that α will be a function of time in layered (reinforced) media.

We examine this by considering the model problem of propagation of heat in a two-layer composite with a regular structure (a representative cross section of the material is shown in Fig. 1) under boundary conditions of the second kind. On the assumption that the thermophysical characteristics of the components do not depend on the temperature, we can write the following for an isolated elementary cross section:

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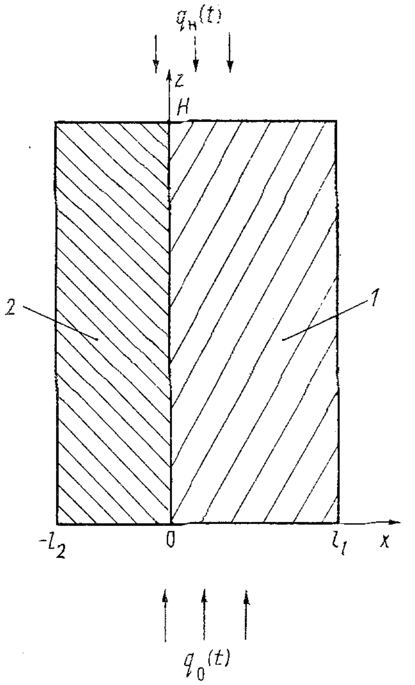

Fig. 1. Representative cross section of a two-layer composite with a regular structure: 1) first layer; 2) second layer; H is the thickness of the material.

$$\lambda_{zi}T_{i,zz} + \lambda_{xi}T_{i,xx} = C_i T_{i,t},\ i = 1,\ 2, \tag{3a}$$

$$T_i(x,\ z,\ 0) = 0, \tag{3b}$$

$$\lambda_{zi}T_{i,z}|_{z=0} = -q_0(t),\ \lambda_{zi}T_{i,z}|_{z=H} = q_H(t), \tag{3c}$$

$$T_{i,x}|_{x=l_i} = 0, \tag{3d}$$

$$\lambda_{x1}T_{1,x}|_{x=0} = \lambda_{x2}T_{2,x}|_{x=0},\ (T_1 - T_2)|_{x=0} = -R_T\lambda_{x1}T_{1,x}|_{x=0}. \tag{3e}$$

We set $\lambda_{z1} > \lambda_{z2}$, i.e., component (1) has better thermal conductivity. Applying the Laplace transformation with respect to time and the Fourier cosine transformation with respect to the temperatures of the components $T_i(x,\ z,\ t)$ (3), we can reduce the equations to a system of ordinary differential equations of the second order in the transform of the temperatures $\tilde{T}_i(x,\ n,\ p)$, whose solution has the form

$$\tilde{T}_i = Q_L\left(\frac{1}{\lambda_{xi}\varphi_i} - (-1)^i M_i(x,\ n,\ p)\right),\ i = 1,\ 2, \tag{4}$$

where

$$\tilde{T}_i(x,\ n,\ p) = \int_0^H \left\{\int_0^\infty T(x,\ z,\ t)\exp(-pt)\,dt\right\}\cos(\psi_n z)\,dz;$$

$$Q_L = \int_0^\infty Q\exp(-pt)\,dt,\ Q = q_0(t) + (-1)^n q_H(t);$$

$$\varphi_i = \frac{p + a_{zi}\psi_n^2}{a_{xi}};\ \psi_n = \frac{n\pi}{H};\ K = (C_1 - C_2)p + (\lambda_{z1} - \lambda_{z2})\psi_n^2;$$

$$M_i = \frac{\mathrm{sh}(\sqrt{\varphi_j}l_j)\,\mathrm{ch}(\sqrt{\varphi_i}[l_i - |x|])\,\lambda_{xj}\sqrt{\varphi_j}K}{\Pi_R\lambda_{x1}\lambda_{x2}\varphi_1\varphi_2},\ j,\ i = 1,\ 2,\ i \neq j;$$

$$\Pi_R = \lambda_{x2}\sqrt{\varphi_2}\,\mathrm{ch}(\sqrt{\varphi_1}l_1)\,\mathrm{sh}(\sqrt{\varphi_2}l_2) + \lambda_{x1}\sqrt{\varphi_1}\,\mathrm{ch}(\sqrt{\varphi_2}l_2)\,\mathrm{sh}(\sqrt{\varphi_1}l_1) + \lambda_{x1}\lambda_{x2}R_\text{т}\sqrt{\varphi_1\varphi_2}\,\mathrm{sh}(\sqrt{\varphi_2}l_2)\,\mathrm{sh}(\sqrt{\varphi_1}l_1);\quad a_{ij} = \lambda_{ij}/C_j.$$

The functions $M_i(x, n, p)$ $(i = 1, 2)$ characterize the heat transfer between the layers; $M_i(x, n, p)$ $(i = 1, 2) \to 0$ as $R_T \to \infty$. To obtain the original components of the temperatures we use the theorem of convolution of the product of the transforms [5] and the formula for the inverse Fourier cosine transforms

$$T_i(x, z, t) = \frac{1}{H}\sum_{n=0}^{\infty} w_n \left\{\int_0^t \widehat{T}_i(x, n, t-\tau)\,Q(\tau)\,d\tau\right\}\cos(\psi_n z), \tag{5}$$

where

$$\widehat{T}_j(x, n, t) = \frac{1}{2\pi i}\int_{\substack{\sigma-i\infty\\(\sigma>p)}}^{\sigma+i\infty} \exp(pt)\left\{\frac{1}{\lambda_{xj}\varphi_j} - (-1)^j M_j\right\}dp;$$

$$w_n = \begin{cases}1, & n = 0,\\ 2, & n = 1, 2, \ldots\end{cases}$$

Having carried out the inverse Laplace transformation, using the theorem of expansion of transforms [5], in much the same way as in [8] we obtain

$$\hat{T}_i(x, n, t) = \begin{cases}\dfrac{1}{C_\text{ef}} - (-1)^i\displaystyle\sum_{k=1}^{\infty} T_{0k}^i E_{0k}, & n = 0,\\ \displaystyle\sum_{m=1}^{A_m} T_i^{nm}E_{nm} + (-1)^i\sum_{k=1}^{\infty} T_{nk}^i E_{nk}, & n = 1, 2, 3, \ldots,\end{cases} \tag{6}$$

where

$$T_{nk}^i = K(p_{nk})\frac{\sin(\alpha_j l_j)\cos\{(l_i - |x|)\alpha_i\}}{\lambda_{xi}\alpha_i^2\alpha_j\Pi_p(\alpha)},\quad i,\ j = 1, 2,\ i \neq j,$$

$$k = 1, 2, 3, \ldots,\ n = 0, 1, 2, \ldots,$$

$$T_1^{nm} = K(p_{nm})\frac{\sin(g_2 l_2)\,\mathrm{ch}(g_1(l_1 - x))}{\lambda_{x1}g_1^2 g_2\Pi_p(g)};\quad E_{ij} = \exp(-P_{ij}t),$$

$$T_2^{nm} = K(p_{nm})\frac{\mathrm{sh}(g_1 l_1)\cos(g_2(l_2 - |x|))}{\lambda_{x2}g_1 g_2^2\Pi_p(g)},\quad C_\text{ef} = (C_1 l_1 + C_2 l_2)/l_0,$$

$$K(p_{kn}) = \psi_n^2(\lambda_{z1} - \lambda_{z2}) - p_{kn}(C_1 - C_2),\ n,\ k = 1, 2, \ldots;$$

$$K(p_{0k}) = p_{0k}(C_1 - C_2),\ k = 1, 2, \ldots;\ l_0 = l_1 + l_2;$$

$$\lambda_\text{ef} = (\lambda_{z1}l_1 + \lambda_{z2}l_2)/l_0,\ a_\text{ef} = \lambda_\text{ef}/C_\text{ef},\ \alpha_i^2 = (p_{nk} - a_{zi}\psi_n^2)/a_{xi},\ i = 1, 2;$$

$$g_1^2 = (\psi_n^2 a_{z1} - p_{nm})/a_{x1},\ g_2^2 = \frac{p_{nm} - a_{z2}\psi_n^2}{a_{x2}},$$

$$2\Pi_p(\alpha) = 2\left.\frac{\partial \Pi_R}{\partial p}\right|_{p=-p_{nk}} = \cos(\alpha_1 l_1)\cos(l_2\alpha_2)[C_2 l_2 + C_1 l_1] -$$

$$-\sin(\alpha_1 l_1)\sin(\alpha_2 l_2)\left[\frac{\lambda_{x2}l_1\alpha_2}{a_{x1}\alpha_1} + \frac{\lambda_{x1}l_2\alpha_1}{a_{x2}\alpha_2} + R_\text{т}C_1C_2 \times\right.$$

$$\left.\times\left(\frac{\alpha_1 a_{x1}}{\alpha_2} + 2\frac{\alpha_2 a_{x2}}{\alpha_1} - R_\text{т}\alpha_1\alpha_2\lambda_{x2}l_2 a_{x1}\right)\right] + \cos(\alpha_1 l_1)\sin(\alpha_2 l_2) \times$$

$$\times\left(\frac{C_2}{\alpha_2} - \frac{\lambda_{x2}\alpha_2}{a_{x1}\alpha_1^2} + R_\text{т}\alpha_2\lambda_{x2}\{C_2 l_2 - C_1 l_1\}\right),$$

$$2\Pi_p(g) = 2\frac{\partial \Pi}{\partial p}\Big|_{p=-p_{nm}} = \mathrm{ch}(g_1 l_1)\cos(g_2 l_2)[C_2 l_2 + C_1 l_1] +$$

$$+ \mathrm{sh}(g_1 l_1)\sin(g_2 l_2)\left\{\frac{\lambda_{x1} l_2 g_1}{a_{x2} g_2} - \frac{\lambda_{x2} l_1 g_2}{a_{x1} g_1} + R_{\mathrm{T}}\frac{\lambda_{x1} g_1 C_2}{g_2} \times\right.$$

$$\left.\times (1 + R_{\mathrm{T}}\lambda_{x2} g_2^2 l_2)\right\} + \mathrm{ch}(g_1 l_1)\sin(g_2 l_2)\left\{\frac{C_2}{g_2} + \frac{\lambda_{x2} g_2}{a_{x1} g_1^2} + R_{\mathrm{T}}\lambda_{x2} g_2 (C_2 l_2 - C_1 l_1)\right\}.$$

The roots p_{nm} are determined from the equation (A_m roots for each n)

$$\lambda_{x1} g_1 \cos(g_2 l_2)\,\mathrm{sh}(g_1 l_1) - \lambda_{x2} g_2 \sin(g_2 l_2)\,\mathrm{ch}(g_1 l_1) = \lambda_{x1}\lambda_{x2} R_{\mathrm{T}} g_1 g_2\,\mathrm{sh}(g_1 l_1)\sin(g_2 l_2),\ 1 \leqslant m \leqslant A_m, \tag{7a}$$

and the roots p_{nk} are determined from the trigonometric equation

$$\lambda_{x2}\alpha_2 \sin(\alpha_2 l_2)\cos(\alpha_1 l_1) + \lambda_{x1}\alpha_1\cos(\alpha_2 l_2)\sin(\alpha_1 l_1) = \lambda_{x1}\lambda_{x2} R_{\mathrm{T}}\alpha_1\alpha_2 \sin(\alpha_1 l_1)\sin(\alpha_2 l_2),\ 1 \leqslant k < \infty. \tag{7b}$$

We note that $p_{nk} < p_{nk+1}$, $p_{nm} < p_{nm+1}$, $a_{z2}\psi_n^2 < p_{nm} < a_{z1}\psi_n^2$, and $a_{z1}\psi_n^2 < p_{nk}$ and, therefore, $p_{nm} < p_{nk}$. The coefficient A_m depends on both the number $n \geq 1$ and on the thermophysical and geometric parameters of the components of the composite.

The integral of the convolution of the functions in (5) can be calculated analytically only for some simple functions Q(t); generally numerical integration must be used to determine it.

We make the boundary conditions more precise to ascertain the effect of the thermophysical and geometric parameters of the layers of the composite on α. Suppose that a pulsed thermal flux acts on the front surface of the material (z = 0) while the back surface (z = H) is thermally insulated. This problem is of purely practical interest as well since is simulates conditions that occur in the pulsed method of determining the thermal-conductivity coefficient ("burst" method) of a layered composite [6]. We write $q_0(t)$ as

$$q_0(t) = \begin{cases} q_0, & 0 \leqslant t \leqslant t_{\mathrm{p}}, \\ 0, & t > t_{\mathrm{p}} \end{cases} \tag{8}$$

In this case the integral in (5) is calculated analytically and the solution of problem (3) for boundary conditions (8) can be written as

$$\Theta_i = 1 + \frac{C_{\mathrm{ef}}}{t_{\mathrm{p}}}\left\{(-1)^{i+1}\sum_{k=1}^{\infty} T_{0k}^i E_{\mathrm{p}}(p_{0k}) + 2\sum_{n=1}^{\infty}\left[\sum_{m=1}^{A_m} T_i^{nm} E_{\mathrm{p}}(p_{nm}) + (-1)^i \sum_{k=1}^{\infty} T_{nk}^i E_{\mathrm{p}}(p_{nk})\right]\cos(\psi_n z)\right\}, \tag{9}$$

where

$$E_{\mathrm{p}}(p_{ij}) = (\exp(p_{ij} t_{\mathrm{p}}) - 1)/(p_{ij})E_{ij}(p_{ij});\ \Theta_i = T_i/T_m;\ T_m = q_0 t_{\mathrm{p}}/(HC_{\mathrm{ef}}),$$

the coefficient α is determined from (1) with allowance for the fact that

$$q^* = -\lambda_{x1}\Theta_{i,x}|_{x=0},\ \hat{\Theta}_i = \frac{1}{l_i}\int_0^{l_i}\Theta_i dx.$$

A program written in FORTRAN IV was used on an EC-1045 computer to calculate Θ_i, q^*, $\hat{\Theta}_i$ and α. The average computing time was a few seconds. We note that in the calculations the number of terms of the series $A_m(n)$ varied from 1 (for $n \leq 3$) to 5-6 (for n = 8).

As an example we considered a carbon composite with the thermophysical characteristics λ_{z1} = 240 W/m·K, $\lambda_{z2} = \lambda_{x1} = \lambda_{x2}$ = 30 W/m·K, and $C_1 = C_2 = 3.2\cdot 10^6$ J/m³·K and the geometric parameters $\ell_1 = \ell_2$ = 0.0006 m and H = 0.003 m. We denote this set of parameters by $K_m(0)$. We use $K_m(0,\ f = y)$ to denote a composite differing from $K_m(0)$ by the value of the parameter f = y. The laser pulse length was chosen at 10^{-4} sec. All of the calculations were carried out for the point z = H, i.e., for the back surface relative to the action of the pulse.

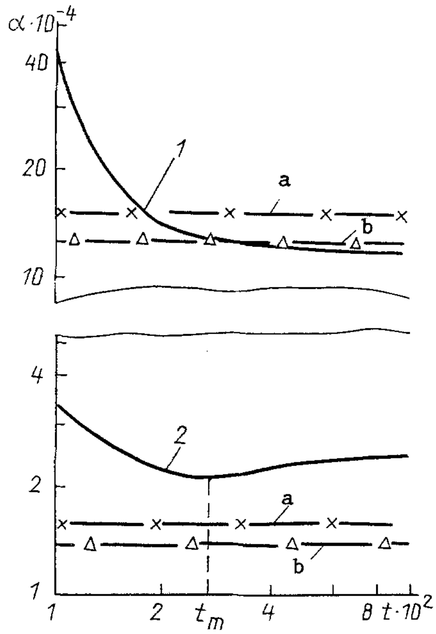

Fig. 2. Dynamics of change of coefficient of heat transfer between components α, $W/m^2 \cdot K$, for the composites: 1) $K_m(0, \lambda_{x1} = 150$ W/m·K); 2) $K_m(0, \lambda_{x1} = 3$ W/m·K). The values of α calculated from (2): a) α_2; b) α_3.

In Fig. 2 the functions α(t) for the materials $K_m(0, \lambda_{x1} = 150$ W/m·K) and $K_m(0, \lambda_{x1} =$ 3 W/m·K) are compared with the values α_2 and α_3 calculated from (2). The results graphically demonstrate unsteady behavior of α(t) as the heat pulse propagates through the material, especially at short times. Moreover, the behavior of α(t) as a function of λ_{x1} qualitatively different: a local minimum of α(t) exists at $\lambda_{x1} = 3$ W/m·K but is absent at $\lambda_{x1} = 150$ W/m·K. Generalizing, we can note that the existence of an extremum of α(t) is characteristic of composites with a low heat-transfer coefficient. The time t_m when the minimum appears does not depend on the amplitude of the laser pulse, i.e., is an intrinsic characteristic of the composite. The value of t_m can be obtained from (1) if we set its first derivative with respect to t equal to zero:

$$q^*_{,t}(\hat{T}_1 - \hat{T}_2) - q^* \times (\hat{T}_{1,t} - \hat{T}_{2,t}) = 0,\ t = t_m.$$

Analysis of Fig. 2 indicates that $\alpha \to \alpha_0$ as $t \to \infty$. After setting an "error" interval, we can determine the corresponding t_a: $t_a = \min\{t,\ \alpha \in [\alpha_a \pm \Delta\alpha]\}$. Then $\alpha = \alpha_a =$ const to within Δα in the interval $t_a \le t < \infty$. The parameter t_a depends on the thermophysical and geometric parameters of the composite as well as on the "error" Δα.

We assess the effect of the boundary conditions on the form of α_a. Since we are interested in the behavior of α as $t \to \infty$, we assign the thermal flux at the boundary z = 0 in the form $q = q_0 \exp(-bt)$ and assume that the surface z = H is thermally insulated. The integral of the convolution of functions (5) can then be calculated analytically

$$T_i(x,\ z,\ t) = \frac{1}{H}\left[\frac{1-\exp(-bt)}{bC_{ef}} - (-1)^i \sum_{k=1}^{\infty} T^i_{0k}\Delta E_{0k} + 2\sum_{n=1}^{\infty}\left\{\sum_{m=1}^{A_m} T_i^{nm}\Delta E_{nm} + (-1)^i \sum_{k=1}^{\infty} T^i_{nk}\Delta E_{nk}\right\}\cos(\psi_n z)\right], \quad (10)$$

where

$$\Delta E_{ij}(t) = (\exp(-bt) - \exp(p_{ij}t))/[p_{ij} - b].$$

The time dependence of the terms in (10) is determined by $\Delta E_{ij}(t)$. As $t \to \infty$ the main contribution will be made by terms with the smallest exponent. As was shown earlier, the inequality $p_{1m=1} < p_{nm} < p_{nk}$; n, k = 1, 2, 3, ...; m = 1, 2, ..., A_m holds. Only $p_{1m=1}$, p_{01},

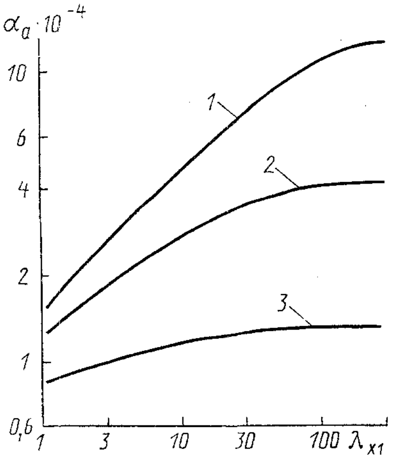

Fig. 3

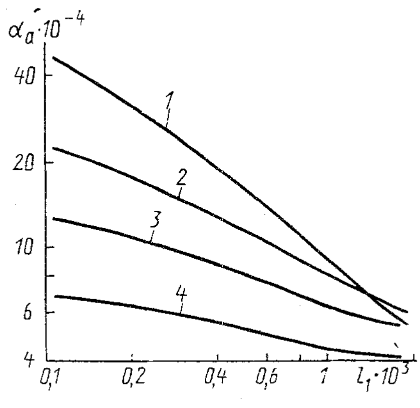

Fig. 4

Fig. 3. The heat-transfer coefficient α_a(W/m^2·K) between the components versus the thermal conductivity coefficient λ_{x1} (W/m·K) at λ_{x2} values of: 1) 30, 2) 10, and 3) 3 W/m·K.

Fig. 4. The thermal-transfer coefficient α_a between the components versus the half-width ℓ_1 (m) of the first layer at ℓ_2 values of: 1) 10^{-4}, 2) $3\cdot10^{-4}$, 3) $6\cdot10^{-4}$, 4) $1.2\cdot10^{-3}$ m.

b, therefore, can be the smallest exponents. We denote $p_{1m=1} = p_1$ and introduce $p_s = \min(p_1, p_{01})$. Two fundamentally different cases are possible, depending on the characteristic of the boundary conditions b under consideration.

<u>1. "Weak" Effect.</u> Suppose that $p_s < b$; then as $t \to \infty$ it follows from (10) that

$$T_i \xrightarrow[t\to\infty]{} \frac{1}{H}\left\{\frac{1}{bC_{\text{ef}}} - (-1)^i T_{01}^i \frac{E_{01}}{b-p_{01}} + 2T_i^{11}\frac{E_{11}}{b-p_1}\cos\left(\frac{\pi z}{H}\right)\right\}. \tag{11}$$

We have considerable possible alternatives. Suppose that $C_1 = C_2$. Bearing in mind that $T^i{}_{01} \sim K(p_{01}) \sim (C_1 - C_2)$, from (6) we have

$$(\hat{T}_1 - \hat{T}_2) \xrightarrow[t\to\infty]{} \frac{2}{H}(\hat{T}_1^{11} - \hat{T}_2^{11})\frac{E_{11}}{b-p_1}\cos\left(\frac{\pi z}{H}\right),$$

$$q^* \xrightarrow[t\to\infty]{} -(2\lambda_{x1}/H)(T_{1,x}^{11}|_{x=0})\frac{E_{11}}{b-p_1}\cos\left(\frac{\pi z}{H}\right)$$

and using (1) we obtain

$$\alpha_{a1} = \left[\frac{l_1 l_2}{l_0}\right]\frac{C_1 C_2}{C_{\text{ef}}}\frac{(p_1 - a_{z2}\psi_1)(\psi_1 a_{z1} - p_1)}{(p_1 - \psi_1 a_{\text{ef}})}. \tag{12}$$

We can say that α_a does not depend on the form of the boundary conditions and is an intrinsic characteristic of the composite, being a function of only its thermophysical and geometric parameters. We note that the time t_a taken by $\alpha(t)$ to reach its asymptotic form α_a depends only on the boundary conditions.

Suppose that $C_1 \neq C_2$. Then either $p_{01} < p_1$ or $p_1 < p_{01}$. In the latter case the derivations above hold and the result is (12). If $p_{01} < p_1$ then as $t \to \infty$ it follows from (11) that

$$(\hat{T}_1 - \hat{T}_2) \xrightarrow[t\to\infty]{} \frac{1}{H}(\hat{T}_{01}^1 + \hat{T}_{01}^2)\frac{E_{01}}{b-p_{01}},\quad q^* \xrightarrow[t\to\infty]{} -\frac{\lambda_{x1}}{H}(\hat{T}_{01,x}^1|_{x=0})\frac{E_{01}}{b-p_{01}},$$

and, using (1), we have

$$\alpha_{a2} = \left(\frac{l_1 l_2}{l_0}\right)\left(\frac{C_1 C_2}{C_{\text{ef}}}\right)p_{01}. \tag{13}$$

On the basis of the thermophysical parameters of actual composites we note that in most cases α_{a1} is the asymptotic value of $\alpha(t)$. We make a numerical analysis of Eq. (12). Figure 3 shows the $\alpha_{a1}(\lambda_{x1})$ curves for three materials: $K_m(0, \lambda_{x2} = 30$ W/m·K, λ_{x1} = var), $K_m(0, \lambda_{x2} = 10$ W/m·K, λ_{x1} = var), and $K_m(0, \lambda_{x2} = 3$ W/m·K, λ_{x1} = var). Analysis of (2) at $\ell_1 = \ell_2 = \ell$ shows that α_2 and α_3 are symmetric with respect to the interchange $\lambda_{x1} \rightleftarrows \lambda_{x2}$

$$\alpha_2 = \frac{2\sqrt{3}}{l}\left(\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\right),\quad \alpha_3 = \frac{3}{l}\left(\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}\right).$$

In actual fact, however, λ_{x1} and λ_{x2} play a slightly different role in the heat transfer. Indeed we consider $\varepsilon(\lambda_{x1}; \lambda_{x2}) = \alpha(\lambda_{x1}; \lambda_{x2})/\alpha(\lambda_{x2}; \lambda_{x1})$; then from Fig. 3 it follows that $\varepsilon(10; 3) = 0.68$, $\varepsilon(30; 10) = 0.76$, and $\varepsilon(30; 3) = 0.73$. Thus, when $\lambda_{x1} > \lambda_{x2}$ the interchange $\lambda_{x1} \rightleftarrows \lambda_{x2}$ causes α_{a1} to decrease by an amount ε which depends on the initial values of the radial thermal conductivities of the components. A 300-fold change in λ_{x1}, from 1 to 300 W/m·K, causes the α_a of the materials $K_m(0, \lambda_{x2} = 3$ W/m·K) and $K_m(0, \lambda_{x2} = 30$ W/m·K) to increase by a factor of 1.6 and 8, respectively.

Figure 4 shows the graphs of α_{a1} versus the half-width of the first layer at different half-thicknesses of the second layer. The coefficient α_{a1} increases as ℓ_1 decreases, the increase being larger for smaller values of ℓ_2. At $\ell_1 > 5$ mm the time t_a taken by $\alpha(t)$ to attain its asymptotic form α_{a1} tends to infinity and in fact the asymptotic value α_{a1} becomes meaningless.

The role of the contact thermal resistance between the layers during heat transfer consists in lowering the rate of heat transfer between the components of the composite. Elsewhere [7] we showed that on the assumption of a linear radial thermal flux density along x we can obtain

$$\alpha(R_{\text{т}}) = \alpha(0)/(1+\alpha(0)R_{\text{т}}). \tag{14}$$

Comparison of (14) with (12) indicates that the approximate formula is practicable and the error of the calculations is less than 0.3% when the thermal resistance varies over the range $0 \le R_T \le 10^{-3}$ K·m²/W. In contrast to the heat-transfer coefficients (2) α_{a1} depends on the thermal conductivity of the components along the z axis, but this dependence is very weak: a six-fold change in $\lambda_{z1}(\lambda_{z2})$ causes α_{a1} to change by 2% (respectively by 0.5%).

2. "Strong" Effect. In this case $b < p_s$ and the behavior of the composite is determined by the boundary conditions and we can write

$$T_i \xrightarrow[t\to\infty]{} \frac{1}{H}\left\{\frac{1-\exp(-bt)}{bC_{ef}} - (-1)^i \sum_{k=1}^{\infty} T_{0k}^i E_b + 2\sum_{n=1}^{\infty}\left[\sum_{m=1}^{A_m} T_i^{nm} E_b + (-1)^i \sum_{k=1}^{\infty} T_{nk}^i E_b\right]\cos(\psi_n z)\right\},$$

where $E_b(t) = \exp(-bt)F(b)$; $F(b) = (p_{ij}-b)^{-1}$. From (1) we obtain

$$\alpha = (\Phi_2 - \Phi_1)/(\lambda_{x1}\Phi_{1,x}|_{x=0}), \tag{14'}$$

where

$$\Phi_i = (-1)^{i+1}\sum_{k=1}^{\infty} T_{0k}^i F(b) + 2\sum_{n=1}^{\infty}\left\{\sum_{m=1}^{A_m} T_i^{nm} F(b) + (-1)^i \sum_{k=1}^{\infty} T_{nk}^i F(b)\right\}\cos(\psi_n z).$$

We note that (14') is valid only for an exponential dependence of the boundary conditions on time; (14') persists but the form of F(b) changes. The constraint due to the thermal insulation of the surface z = H is not fundamental: the form of (12) and (13) does not change if $q_H(t) < \exp(-p_s t)$. When $q_H(t) > \exp(-p_s t)$ we must determine $q(t) = \max_{t\to\infty}\{q_0(t), q_H(t)\}$ and then carry out the calculations (14) for q(t).

In summary, the behavior of $\alpha(t)$ as $t \to \infty$ depends on the sign of the inequality $\exp(-p_s t) \gtrless q(t)$. If the "greater than" sign holds, then the external effect can be said to be "weak" and the asymptotic value α_a is determined by the thermophysical and geometric parameters of the composite; otherwise, $\alpha_a = \alpha(q)$.

The above analysis demonstrated the dynamics of $\alpha(t)$ and made it possible to obtain its asymptotic value as well as the dependence of the latter on the parameters of the composite. When calculating temperature fields in layered (reinforced) media within the framework of the two-temperature approach one must assess the effect of the unsteady nature of α on the accuracy of the calculation and take it into account, if necessary. A similar analysis should also be carried out for reinforced composites.

NOTATION

Here z and x denote the space variables; t is the time; $T_i(x, z, t)$ and $\hat{T}_i(z, t)$ are the temperature of the i-th component and its average cross-sectional value; q_{ij} is the density of the thermal flux from the i-th to the j-th component; α is the coefficient of heat transfer between components; C_i, λ_{xi}, and λ_{zi} are the coefficients of volumetric heat transfer and the radial and axial thermal conductivity, respectively; R_T is the coefficient of contact thermal resistance between layers; ℓ_i is the half-thickness of the layers; q_0 is the thermal flux density of the laser radiation; t_p is the length of the laser burst; p and n are the parameters of the Laplace and Fourier cosine transforms; and ℓ_k is the characteristic size of microinhomogeneities.

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PROBLEM OF HEAT AND MASS TRANSFER DURING SHORT-TIME PHASE CONTACT

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The associated mixed boundary-value problem of multicomponent mutually related heat and mass transfer during short-time contact of two phases with arbitary dimensionalities of the transfer vector potentials in them through a boundary with selective penetrability during excitation of material flows in each of the phases, which are absent in the other phase, is formulated and solved. This is done with the purpose of generalizing the model of phase penetration and restoration in the theory of mass exchange, and of similar models in the theory of heat exchange, based on the phenomenon of short-time contact interaction. The validity limits of these models are estimated. An effect is observed of internal phase flows on the intensity of nonstationary interphase exchange.

The contemporary intense development of material processing technology leads to an enhanced role of nonstationary mutually related exchange processes in comparison with the stationary decoupled ones. This fact is so far not sufficiently reflected in the solution of problems of heat and mass transfer (HMT) at small Fourier numbers, for short-time contact (SC) phases. The physical model concepts have been developed well for both heat- and mass-transfer, but separately. In the theory of mass exchange they are represented by permeation (Higby) and phase

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